STUDY MODULE DESCRIPTION FORM

Name of the module/subject **Stochastic methods and mathematical statistics**		Code **1010342631010347255**
Field of study **Mathematics**	Profile of study (general academic, practical) **general academic**	Year /Semester **2 / 3**
Elective path/specialty **-**	Subject offered in: **Polish**	Course (compulsory, elective) **obligatory**
Cycle of study: **Second-cycle studies**	Form of study (full-time,part-time) **full-time**	
No. of hours Lecture: **30** Classes: **30** Laboratory: **-** Project/seminars: **-**		No. of credits **4**
Status of the course in the study program (Basic, major, other) **other**	(university-wide, from another field) **university-wide**	
Education areas and fields of science and art **the sciences** **Mathematical sciences**		ECTS distribution (number and %) **4 100%** **4 100%**

Responsible for subject / lecturer:

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Prerequisites in terms of knowledge, skills and social competencies:

1	**Knowledge**	Student knows the basic concepts of subject of Probability and statistics.
2	**Skills**	Student has a ability to think logically (student formulates new facts from a previously known facts).
3	**Social competencies**	Student knows the limits of his own knowledge and understands the need for further education.

Assumptions and objectives of the course:

The main goal is to acquaint a student with the basic concepts of the theory of stochastic processes, certain classes of stochastic processes and to gain by the student the ability to calculate some characteristics of those processes. Furthermore the student will get to know the notion of stochastic integral, will acquire the ability to determine the stochastic integrals and stochastic differentials (stochastic calculus), and the ability to examine whether some process is a solution to a given stochastic differential equation.

Study outcomes and reference to the educational results for a field of study

Knowledge:

1. Student understands the construction of mathematical theories, can use mathematical formalism to construct and analyze simple mathematical models, which describe phenomena from other disciplines. - [K_W03]

2. Student knows the basic theorems of the known branches of mathematics. - [K_W04]

Skills:

1. Student uses the term of probability space; is able to build and analyze a mathematical model of scientific experiment of random phenomena. - [K_U30]

2. Student is able to give various examples of discrete and continuous probability distributions, and discuss selected random experiments and mathematical models in which those distributions appear; knows the practical application of basic probability distributions. - [K_U31]

3. Student is able to determine the parameters of distribution function of discrete and continuous random variables; can apply the limit theorems and laws of large numbers to estimate probability of some random variables. - [K_U33]

4. Student is able to use the statistical characteristics of population and their counterparts which can be calculate by using a probability sample. - [K_U34]

Social competencies:

1. Student is able to accurately formulate questions in order to deepen their understanding of given topic or in order to find the missing pieces of reasoning. - [K_K02]

2. Student understands and appreciates the importance of intellectual honesty in the activities of their own and other people; proceeds ethically. - [K_K04]

Assessment methods of study outcomes

Classes:

- Assessment of the knowledge and skills based on tests, which are carried out in the middle of the semester and during the last classes.

Lectures:

- Assessment of the knowledge and skills based on the written exam.

Course description

Basic concepts of the theory of stochastic processes: stochastic process, sample path of stochastic process, interpretation of a stochastic process as a random variable, interpretation of a stochastic process as a function of two variables, measurability of stochastic process, filtration, class of adapted processes, class of indistinguishable processes, modification of the stochastic process, Kolmogorov's continuity theorem, continuity types of the stochastic process (continuity with probability one, continuity in probability, mean-square continuity).

Probabilistic description of the stochastic process: examples of processes which are analytical defined, some characteristics of sums and products of stochastic processes, canonical form of the stochastic process, processes with independent increments, telegraphic signals (synchronous processes, asynchronous processes).

Markov process: definition of Markov process, examples of Markov processes, Markov chain, certain examples of the problems of mass service.

Poisson process: definition of Poisson process, properties of Poisson process.

Some characteristics of differentiable processes and integrable processes (convergence in mean-square sense, mean-square derivative, mean-square integral, ergodicity), and elements of spectral analysis of stationary stochastic processes (spectral density).

Martingales: definitions of discrete-time martingale and continuous-time martingale, examples of martingales, verification whether the process is a martingale, transformation of the known stochastic processes to martingales, Doob's martingale inequality, definition of submartingale and supermartingale, example of submartingale and supermartingale in game theory.

Markov time: definition and properties of Markov time, examples of Markov time, definition and example of the first passage time of stochastic process to the set, definition and properties of stopping time, definition and properties of local martingale.

Brownian motion: definition of Brownian motion, definition of standard Brownian motion, properties of standard Brownian motion, properties of sample path of Brownian motion, brownian motion as a square integrable martingale.

Wiener process: definition of Wiener process, Levy's theorem, examples of Wiener processes, Wiener process as a martingale, examples of stochastic processes determined by transformation of Wiener process, which turn out to be a martingales, example of application of the Wiener process into theory of financial mathematics - geometric Brownian motion (exponential Brownian motion).

Construction of Itô integral: white noise, definition of simple function, comparison of the construction of Itô integral with the construction of Riemanna-Stieltjesa integral, non-anticipating processes, predictable processes, Itô isometry, definition of Itô integral, example of calculation of stochastic integral based on the definition of Itô integral, properties of stochastic Itô integral.

Itô formula: definition of Itô process, definition of stochastic differential, definition of diffusion process, Itô's lemma, application of the Itô formula (calculation of stochastic integrals and stochastic differentials, and verification whether some stochastic process is a solution to a given stochastic differential equation).

Applied methods of education:

- lectures - theory presented in connection with the current knowledge of students,

- laboratories - computational experiments.

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Basic bibliography:

1. A. Plucińska, E. Pluciński, Probabilistyka: statystyka matematyczna, procesy stochastyczne, rachunek prawdopodobieństwa, Wydawnictwo Naukowe PWN SA, Warszawa 2017.

2. A. Pieniążek, J. Weiss, A. Winiarz, Procesy stochastyczne w problemach i zadaniach, Wydaw. Politechniki Krakowskiej im. Tadeusza Kościuszki, Kraków 2000.

3. R. Sz. Lipcer, A. N. Sziriajew, Statystyka procesów stochastycznych: filtracja nieliniowa i zagadnienia pokrewne, PWN, Warszawa 1981.

Additional bibliography:

1. B. Oksendal, Stochastic differential equations. An introduction with applications, Springer-Verlag, Berlin Heidelberg New York 2000.

2. Z. Brzezniak, T. Zastawniak, Basic stochastic processes. A course through exercises, Springer-Verlag, London 2002.

3. A. Iwanik, J. K. Misiewicz, Wykłady z procesów stochastycznych z zadaniami. Cz. 1: Procesy Markowa, Oficyna Wydaw. Uniwersytetu Zielonogórskiego, Zielona Góra 2009.

4. M. Kozaryn, M. Michta, K.Ł. Świątek, Stochastic inclusions driven by two-parameter martingales, Dynam. Systems Appl. 25 (2016) 123-152.

Result of average student's workload

Activity	Time (working hours)
1. Participation in lectures	30
2. Participation in classes	30
3. Preparation for each classes	20
4. Preparation for written test	12
5. Assessment classes	4
6. The written exam	4
7. Consultations	4

Student’s workload

Source of workload	hours	ECTS
Total workload	104	4
Contact hours	72	3
Practical activities	30	1